Three spatial dimensions in a relational clock substrate: known criteria in one setting, and a counting consistency condition

Ioannis Kokkinis*

Draft, September 28, 2026

Abstract

We study a relational substrate built from persistent places, each carrying a clock, whose ticks are the events and whose links are formed by coincidences of ticks. In this single setting several known criteria for the dimension of space arise together, each resting on a different physical premise. (A) Stable bound orbits in a potential $\propto r^{2-D}$ require $D < 4$ (Ehrenfest's classic argument). (B) If the gravitational potential is the discounted visit count of random-walk ancestral lineages, it stays finite in the limit of no fading only when those walks are transient, which by Pólya's theorem requires $D \geq 3$; this is equivalent to the familiar non-decay of the Laplacian Green's function for $D \leq 2$, and its novelty lies in the physical premise. (C) If quantum phase is carried by clocks synchronised through local coincidence coupling, a coherent phase over arbitrary distances needs true long-range order, which by the Mermin–Wagner theorem is absent for $D \leq 2$; simulations on lattices and on random graphs of matched degree agree, and a gravity-covariant coupling is needed to keep order under gravitational and quenched rate differences. (D) We show that four static counting assumptions (coincidence metric, multiplicative clock composition, covariant propagation of the gravitational count, and exact additivity of counts) are mutually consistent, for non-trivial gravity, if and only if $D = 3$; the exterior field is then the exponential metric. This last result is a consistency condition closely tied to the known dimension dependence of static reductions of general relativity, not an independent derivation. Given the stated premises, the criteria are jointly satisfied only for $D = 3$. We list the premises, the limitations of each route, and the parts of the gravity sector that remain open or are already excluded by observation.

1 Introduction

Why space has three dimensions is an old question. Ehrenfest observed that planetary orbits and atoms are stable only for three spatial dimensions [1]; Tangherlini extended the analysis to the Schwarzschild and hydrogen problems in D dimensions [2]. Barrow reviewed the many physical and anthropic arguments that depend on dimensionality [3], Tegmark gave a well-known summary in which $3 + 1$ is singled out by stability and predictability requirements [4], and Scargill re-examined the case for $2 + 1$ dimensions [5]. Rabinowitz discusses further arguments [6]. In quantum-gravity approaches the dimension is itself dynamical: in causal dynamical triangulations the large-scale spectral dimension is close to four while it is reduced at short scales [7].

Most of these arguments take a fixed dynamical law (Newtonian gravity, the Schrödinger equation, general relativity) and ask in which D it gives an acceptable world. Here we take a minimal relational substrate in which space, time and gravity are defined by counting events, and ask which known dimension criteria it inherits. The purpose is not to present new independent derivations of $D = 3$, but to show that in one setting several criteria—bound orbits, localised gravity, coherent quantum phase—apply to the same objects, and to add one condition specific to the substrate: the consistency of its static counting rules.

The routes are not of equal standing. Route A is classical. Routes B and C apply known theorems (Pólya's recurrence theorem and the Mermin–Wagner theorem) to mechanisms of the substrate; their content lies in the premise that the mechanism is the one realised in nature, and each has limitations that we state. Route D is a short theorem whose mathematics is elementary and whose content is in its physical assumptions, one of which (exact additivity) is strong. We collect the premises and open problems in Sec. 7.

2 The substrate

The model has two levels of ontology.

1. *Places.* A set of persistent places, each with its own clock. Places are neither created nor destroyed. A clock ticks at a local rate λ (the lapse), measured against a common substrate time t.

*info@upg.gr

2. *Events.* Each tick of a place is an event. Events inherit from earlier events (at the same place or at neighbouring places), so every event has an ancestry.

Links between neighbouring places are formed by *coincidences*: a link is an interaction, and an interaction requires both ends to be active together. The rate of a link between places s and t is therefore proportional to the product $\lambda_s\lambda_t$.

Light. A signal propagates along links, and light is identified with the first-arrival front of such propagation. On statistically isotropic substrates the front is isotropic and defines an emergent light cone and a common speed.

Gravity. Matter places tick at an extra rate. Gravity is the *news* of those ticks, propagated through the substrate, and its effect is to slow the clocks that receive it: $\lambda < 1$ near matter. This gives gravitational time dilation directly.

Spatial metric from coincidence. Because link rates are $\propto \lambda^2$ while clocks tick at λ, the coordinate speed of a signal is $\propto \lambda^2$. Matter uses the same links, so its hop rate is $\propto \lambda^2$ while its internal clock runs at λ; its size in substrate coordinates therefore scales as λ, i.e. rulers shrink near mass. The local speed of light, $\lambda^2/(\lambda \cdot \lambda) = 1$, is constant. The physical metric seen by matter is

$$\mathrm{d}s^2 = -\lambda^2\,\mathrm{d}t^2 + \lambda^{-2}\,\delta_{ij}\,\mathrm{d}x^i\mathrm{d}x^j, \tag{1}$$

so that $g_{tt}g_{rr} = -1$. With $\lambda = e^{\Phi}$ this gives PPN parameters $\beta = \gamma = 1$ in the weak field. On a two-dimensional random (Delaunay) graph a plane wave passing a mass was propagated with the coincidence rule and with a single-ended rule (link rate λ_t). The ratio of arrival delays was 1.95–2.01 over six impact parameters, against the predicted factor 2 that distinguishes $\gamma = 1$ from $\gamma = 0$; the ratios of deflections (slopes of the delay) are noisier, 1.66–2.24.

3 Route A: stable orbits require $D < 4$

If the static potential of a point mass obeys a Laplace law in the substrate, it falls as $V(r) = -k\,r^{-(D-2)}$ for $D \geq 3$, with $k > 0$. For a test body of angular momentum ℓ (per unit mass) the effective radial potential is

$$V_{\text{eff}}(r) = -\frac{k}{r^n} + \frac{\ell^2}{2r^2}, \qquad n = D - 2. \tag{2}$$

A circular orbit at r_0 satisfies $\ell^2 = nk\,r_0^{2-n}$, and

$$V''_{\text{eff}}(r_0) = \frac{3\ell^2}{r_0^4} - \frac{n(n+1)k}{r_0^{n+2}} = \frac{nk}{r_0^{n+2}}\,(2-n). \tag{3}$$

The orbit is stable only if $n < 2$, i.e. $D < 4$. This is Ehrenfest's argument [1, 2].

In the substrate the r^{2-D} law is not assumed but comes from the lineage mechanism of Route B—exactly only in the limit of no fading, $\epsilon = 0$. For $\epsilon > 0$ the potential is Yukawa-screened with range ξ [Eq. (6)], and stable orbits are then restricted to $r \lesssim \xi$. Observation requires ξ much larger than any bound system, so this restriction is not a practical limitation, but the connection to Route B is exact only at $\epsilon = 0$.

4 Route B: localised gravity from transient lineages, $D \geq 3$

4.1 Genealogical rule

One concrete realisation of the news mechanism uses only inheritance. Every event carries a number s. When place v fires, the new event copies s from *one* parent chosen uniformly at random among its parents—v's own previous event, or the latest event of one of its k neighbours—and updates

$$s_{\text{new}} = (1-\epsilon)\,s_{\text{parent}} + \delta\,[v \in \mathcal{M}], \tag{4}$$

where $\mathcal{M}$ is the set of matter places, ϵ is a fading rate and δ a source strength. The clock rate of a place is $\lambda_v = \lambda_0(1 - \alpha\langle s\rangle_v)$. No potential, Laplacian, or coupling constant G is put in.

Table 1: Genealogical potential s of a single matter place, computed exactly from the lineage Green's function, as the fading rate ϵ decreases (from 0.02 to 0.0003). Values from the research log.

D	quantity	values as ϵ decreases
2	$s(r=4)$	0.24, 0.59, 1.06, 1.60 (grows $\sim \log 1/\epsilon$)
3	$s(r=4)$	0.033, 0.069, 0.099, 0.119 (converges)
3	$s(4)/s(8)$	$\to 2.45$, approaching the Newtonian 2
4	$s(4)/s(8)$	$18.6 \to 8.1$, approaching 4

Following an event's ancestry backwards, each step goes to the same place with probability $1/(k+1)$ or to a uniformly chosen neighbour: the ancestral lineage is a lazy random walk X_j on places, with transition operator P. Unrolling the rule,

$$s(v) = \delta \sum_{j\geq 0} (1-\epsilon)^j \, P_v(X_j \in \mathcal{M}) = \delta \, G_\epsilon(v, \mathcal{M}), \tag{5}$$

the discounted Green's function of the lineage walk, which solves $[\, I - (1-\epsilon)P \,]\, s = \delta \, \mathbf{1}_{\mathcal{M}}$. On the hypercubic lattice the per-coordinate step variance is $\sigma^2 = 2/(2D+1)$ and $P - I \approx (\sigma^2/2)\nabla^2$, so to leading order $(-\nabla^2 + \xi^{-2})\, s \propto \mathbf{1}_{\mathcal{M}}$ with screening length

$$\xi = \sqrt{\sigma^2/(2\epsilon)}. \tag{6}$$

(The research log on which this paper is based wrote $\xi = \sqrt{\sigma^2/\epsilon}$, omitting the factor 1/2 of the diffusion generator; Eq. (6) corrects it.) For $\epsilon \to 0$, $s \propto r^{2-D}$. The potential is linear in the matter indicator, so sources superpose, and the "mass" of a source is its rate of matter events.

Forward test. The event process was simulated on a periodic three-dimensional lattice ($L = 41$, $\epsilon = 0.004$, six independent runs of 6000 time units) and compared with the exact Green's function (5), with no fitted parameter. The ratio simulation/exact is 0.997, 0.995, 1.007, 0.997 at $r = 0, 1, 2, 3$, i.e. within 1% for $r \leq 3$, and 0.971 at $r = 4$. At larger distance it is 0.869, 0.805, 0.801, 0.877, 0.899 at $r = 6, 8, 10, 13, 16$. Although each point is only 1.4–1.8 standard errors from unity, all five lie low by 10–20%; we regard this as a systematic deficit, not noise, and it is at present unexplained.

4.2 Dimension

Premise. Gravity is long-ranged on all observed scales, so the physically relevant regime is $\epsilon \to 0$ (inherited states fade slowly or not at all), and a single mass must produce a finite, localised potential in that limit.

Pólya's theorem [8] states that simple random walks on $\mathbb{Z}^D$ are recurrent for $D \leq 2$ and transient for $D \geq 3$. Since the lineages are random walks, the expected number of visits to a finite matter region, $G_0(v, \mathcal{M})$, is infinite for $D \leq 2$ and finite for $D \geq 3$:

- $D \leq 2$: every lineage revisits the matter region infinitely often. As $\epsilon \to 0$, the potential of a single matter place diverges *everywhere*; one mass would slow every clock without bound.
- $D \geq 3$: the potential stays finite and falls as r^{2-D}.

Exact (FFT) evaluations of Eq. (5) show the transition (Table 1). In $D = 2$ the potential at $r = 4$ grows like $\log(1/\epsilon)$ without bound; in $D = 3$ it converges, and the ratio $s(4)/s(8)$ approaches the Newtonian value 2 from above; in $D = 4$ the ratio approaches 4.

4.3 Scope and limitations

- *Relation to known results.* Since G_ϵ is the resolvent of a discrete Laplacian, the statement is equivalent to the familiar fact that the Laplacian Green's function does not decay (it grows logarithmically or linearly) for $D \leq 2$. The mathematics is therefore standard; what is new, to our knowledge, is the physical premise that the gravitational potential *is* a lineage visit count, which makes Pólya's recurrence the relevant criterion. A literature search found no prior use of Pólya recurrence to argue for $D = 3$, though a search cannot establish novelty.
- *Strength of the $D = 2$ exclusion.* In $D = 2$ the divergence is only logarithmic: at finite fading the potential grows as $\ln \xi$. For any finite ξ (for example, one set by a cosmological fading time) two-dimensional genealogical gravity is finite, and the exclusion holds only in the strict limit $\xi \to \infty$. For $D = 1$ the field is usable only in screened form; a self-consistent $1+1$ simulation of two bodies interacting through it shows mutual attraction with a short-ranged force.

- *Uniform matter.* With a uniform density of matter the limit $\epsilon \to 0$ diverges in $D = 3$ as well (a Seeliger-type divergence of the summed potential). Route B concerns localised sources; a cosmological setting needs either finite ξ or a subtraction of the mean, which is not addressed here.
- *Lattices versus random graphs.* Pólya's theorem is a lattice result. Transience of random walks on the random substrates of Route C (and of Sec. 2) is expected from their large-scale dimension but needs separate support; we have not tested it here.

Combined with Route A, and given its premise, $D = 3$ is the only dimension in which genealogical gravity is both localised and admits stable bound structures.

5 Route C: coherent clock phase requires long-range order, $D \geq 3$

5.1 Why the clock phase must be coherent

Suppose the quantum phase of matter is accumulated from the substrate's own ticks, and that successive tick intervals carry independent relative jitter σ. Each path then accumulates independent phase noise, and the ensemble interference visibility between two branches decays as $V(t) = \exp(-\mu^2\sigma^2 t)$ per lattice step (confirmed in a $1+1$ Dirac walk). In physical units the decoherence rate is $\Gamma = (mc^2/\hbar)^2\sigma^2 t_{\text{tick}}$, and coarse-graining does not change it. Observed matter-wave interference then requires, as an order-of-magnitude bound, $t_{\text{tick}} \lesssim 10^{-55}\,\text{s}$ for large-molecule interferometry, far below the Planck time. Independent random clocks are therefore excluded as the carrier of quantum phase: the phase must be a *coherent* clock hand.

Premise of Route C. The coherence is produced by synchronisation of the place clocks through their own *local* coincidence couplings.

5.2 Mermin–Wagner

With identical clocks and noise, the synchronised state is an XY-type ordered phase whose fluctuations are spin waves. The variance of the phase difference between places a distance r apart grows linearly in $D = 1$, logarithmically in $D = 2$, and saturates in $D = 3$ (exact lattice Green's function at noise ratio $T/J = 1$: from 0.33 at $r = 1$ to 0.50 at $r = 32$ in $D = 3$, versus $0.50 \to 1.62$ in $D = 2$ and $1.0 \to 1008$ for $r = 1 \to 1024$ in $D = 1$). By the Mermin–Wagner theorem [9] a continuous symmetry with short-range interactions cannot be spontaneously broken for $D \leq 2$.

Two qualifications apply. First, the theorem is an asymptotic, equilibrium statement. In $D = 2$ the spin-wave variance grows as $(T/\pi J)\ln r$; at small noise ratio this growth is very slow, and a phase can remain coherent over astronomically large distances. The exclusion of $D = 2$ is therefore quantitative only if the noise ratio is of order one. Second, local diffusive synchronisation is slow: with place spacing near the Planck length it can order regions only of size $\sim \sqrt{c\, l_{\text{P}}\, t}$, about $50\,\mu\text{m}$ over the age of the universe. Macroscopic clock coherence therefore cannot be built up by local synchronisation after the fact; it would have to be inherited, for example from an ordered nucleation of the substrate. The dimension criterion then applies to the *stability* of an inherited ordered phase against noise rather than to its formation, which weakens this route.

5.3 Simulations

Model. Each place carries a phase θ (common frequency removed) with diffusive jitter of strength $\mathcal{D}_\theta$. Neighbours couple only at random coincidence events: in each step each link fires with probability $p = 0.5$ and then pulls both phases together by $\varepsilon \sin \Delta\theta$. The mean coupling is $K = p\varepsilon$. Runs start ordered, and the order parameter $m = |\langle e^{i\theta}\rangle|$ is averaged over the second half of the run.

Lattices ($\varepsilon = 0.4$, $\mathcal{D}_\theta/K = 0.7$, chosen below the two-dimensional quasi-order threshold to give $D = 2$ its best chance). In $D = 1$ the order decays as $m \propto L^{-0.5}$; in $D = 2$ it decays as a slow power law, $m \sim L^{-0.09}$ (quasi-long-range order); in $D = 3$ it is constant within 0.008 over a factor three in linear size (Fig. 1, Table 2).

Random places ($\varepsilon = 0.12$, same noise ratio). Poisson-random places in a periodic box, each linked to its six nearest neighbours, giving mean degree about 7 in both $D = 2$ and $D = 3$. Connectivity is thus matched and only the dimension differs. The result is the same as on lattices: m keeps falling in $D = 2$ and is constant in $D = 3$.

5.4 Gravity and disorder require a covariant coupling

Gravity makes clocks run at different rates, $\lambda(x)$, while synchronisation pulls them together. With plain coupling,

$$\dot{\theta}_i = \omega\lambda_i + K \sum_j \sin(\theta_j - \theta_i), \tag{7}$$

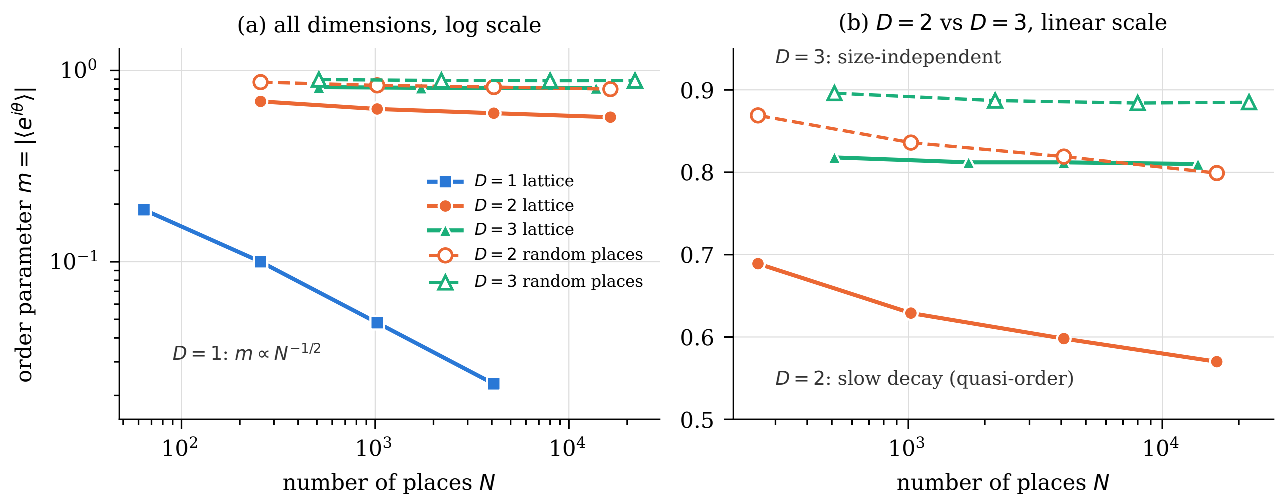

Figure 1: Order parameter of coincidence-synchronised clocks versus number of places N. Filled markers and solid lines: hypercubic lattices ($N = L^D$). Open markers and dashed lines: Poisson-random places with six-nearest-neighbour links (mean degree about 7 in both dimensions). (a) Log scale, all dimensions; the $D = 1$ data follow $m \propto N^{-1/2}$. (b) Linear scale for $D = 2$ and $D = 3$: the $D = 2$ order decays slowly with size on both substrates, while the $D = 3$ order is size-independent. Coupling strengths differ between the lattice and random-place runs, so absolute levels should not be compared across substrates.

on a $12 \times 12 \times 24$ lattice with $\lambda = 1 + gz$ (open in z), the frequency gradient measured in units of the full redshift ωg is -0.002 for $g = 0.002$: the clocks lock and the redshift is erased. For $g = 0.02$ it is 0.883, the redshift is 12% short, and phase slips (random 2π jumps) collapse the comoving order to 0.10. A continuum locking estimate gives the locking length $\ell_{\text{lock}} = \sqrt{8K/(\omega g)}$; plain synchronisation either erases redshift (below ℓ_{lock}) or produces slips (above it).

A *covariant* coupling compares hands after allowing for their expected difference,

$$\dot{\theta}_i = \omega\lambda_i + K \sum_j \sin(\theta_j - \theta_i - A_{ij}), \qquad \dot{A}_{ij} = \omega(\lambda_j - \lambda_i), \tag{8}$$

with a local bond variable A_{ij}. Then $\theta_i = \omega\lambda_i t$ is an exact solution, and noise acts around it as in flat space. In the same test the frequency gradient is 1.001 ($g = 0.002$) and 1.000 ($g = 0.02$) of the full redshift, with comoving order 0.915, equal to the flat-space value. In this form the lapse enters the clock network as the time component of a U(1) connection on the clock phases.

The same coupling is needed for quenched disorder. With random natural rates $\lambda_i = 1 + \sigma\xi_i$ on a three-dimensional lattice, plain coupling loses phase order as L grows (Table 2), consistent with the known absence of phase order for oscillators with random frequencies below four dimensions [10, 12]; frequency entrainment survives in lower dimensions [11]. Covariant coupling absorbs any rate differences, random or gravitational, and keeps the comoving order size-independent in $D = 3$.

5.5 Caveat: the route assumes local coupling

The Mermin–Wagner theorem requires short-range interactions. We tested this by adding random long-range links to two-dimensional random graphs (last rows of Table 2; noise ratio $\mathcal{D}_\theta/K = 1.0$, one seed per point). Without long links the order falls from 0.775 to 0.702 and then stays at 0.703 over the next factor of four in N, so with a single seed this run alone does not establish continued decay (the decay is clearer in the multi-size runs of Fig. 1). With 2% added links the order is nearly constant; with 10% it is higher and constant. The data are consistent with the expectation that a finite density of non-local clock-coupling links restores order in $D = 2$. Route C therefore holds only if the links that couple clocks are local: any extension of the substrate that introduces non-local relations must keep them from coupling clocks, or give them vanishing density.

6 Route D: Kokkinis' theorem (static counting closure)

Setting. A substrate of places with D spatial dimensions and substrate coordinates x^i; static configurations only. The clock rate (lapse) at a place is λ. K is the news count arriving at a place (the number of news events per unit substrate time).

Assumptions.

Table 2: Simulated order parameters. Lattice and random-place rows: m of θ. Disorder rows: for plain coupling, m of θ itself; for covariant coupling, m of the comoving phase $\theta_i - \omega\lambda_i t$. Long-link rows: noise ratio $\mathcal{D}_\theta/K = 1.0$, a single seed per point; the percentage is the number of added random links relative to the number of places. All values from the research log.

system	sizes	order
lattice, $D = 1$	$L = 64, 256, 1024, 4096$	0.187, 0.100, 0.048, 0.023
lattice, $D = 2$	$L = 16, 32, 64, 128$	0.689, 0.629, 0.598, 0.570
lattice, $D = 3$	$L = 8, 12, 16, 24$	0.818, 0.812, 0.812, 0.810
random places, $D = 2$	$N = 256, 1024, 4096, 16384$	0.869, 0.836, 0.819, 0.799
random places, $D = 3$	$N = 512, 2197, 8000, 21952$	0.896, 0.887, 0.884, 0.885
$D = 3$ disorder $\sigma = 0.08$, plain (θ)	$L = 8, 16, 24$	0.913, 0.910, 0.895
$D = 3$ disorder $\sigma = 0.08$, covariant (comoving)	$L = 8, 16, 24$	0.922, 0.920, 0.919
$D = 3$ disorder $\sigma = 0.15$, plain (θ)	$L = 8, 16, 24$	0.897, 0.891, 0.836
$D = 3$ disorder $\sigma = 0.15$, covariant (comoving)	$L = 8, 16, 24$	0.921, 0.919, 0.919
$D = 2$ random, 0% long links	$N = 1024, 4096, 16384$	0.775, 0.702, 0.703
$D = 2$ random, 2% long links	$N = 1024, 4096, 16384$	0.775, 0.774, 0.766
$D = 2$ random, 10% long links	$N = 1024, 4096, 16384$	0.826, 0.817, 0.814

(A1) Coincidence. Links form at rate $\lambda_s\lambda_t$, so the spatial metric is $h_{ij} = \lambda^{-2}\delta_{ij}$ and the physical metric is $g = -\lambda^2 \mathrm{d}t^2 + \lambda^{-2}\mathrm{d}x^2$ [Eq. (1)].

(A2) Multiplicative composition (motivated by the equivalence principle). $\lambda = F(K)$, with F continuous, $F(0) = 1$ and $F(K_1 + K_2) = F(K_1)F(K_2)$: a clock that is already slowed responds to additional news in the same proportion as an unslowed one.

(A3) Covariance. News propagates on the emergent spacetime, so in vacuum the static news count obeys $\partial_i\big(\sqrt{-g}\, g^{ij}\partial_j K\big) = 0$.

(A4) Additivity. Counts add exactly: if K_1 and K_2 are the vacuum news fields of two separate sources, then $K_1 + K_2$ is the vacuum news field of both together.

Theorem 1 (Kokkinis' theorem[1])**.** *Assume non-trivial gravity, i.e. $F \not\equiv 1$. Then (A1)–(A4) are mutually consistent if and only if $D = 3$. In that case $\lambda = e^{sK}$ with $s \neq 0$, K is harmonic in substrate coordinates, and the exterior of a point source of mass M is the exponential metric*

$$\mathrm{d}s^2 = -e^{-2M/r}\,\mathrm{d}t^2 + e^{2M/r}\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right), \tag{9}$$

which has no horizon and a throat at $r = M$.

Proof. (i) By (A2), F solves Cauchy's multiplicative equation and is continuous, so $F(K) = e^{sK}$ for some constant s; non-trivial gravity means $s \neq 0$.

(ii) By (A1), $\sqrt{-g} = \lambda \cdot \lambda^{-D}$ and $g^{ij} = \lambda^2\delta^{ij}$, so $\sqrt{-g}\, g^{ij} = \lambda^{3-D}\delta^{ij} = e^{(3-D)sK}\delta^{ij}$. (A3) then becomes

$$\partial_i\Big(e^{(3-D)sK}\,\partial_i K\Big) = 0. \tag{10}$$

(iii) $D = 3$: the equation is Laplace's equation, which is linear, so (A4) holds. The spherical solution is $K = -M/(sr) + \text{const}$, and with $\lambda = e^{sK}$ this gives Eq. (9). Horizons need $\lambda = 0$, but $\lambda = e^{-M/r} > 0$ for all $r > 0$. The areal radius $R = r\,e^{M/r}$ has its minimum at $r = M$ (the throat, $R = eM$).

(iv) $D \neq 3$: since $s \neq 0$, put $u = e^{(3-D)sK}$. The equation becomes Laplace's equation for u, so vacuum news fields are exactly $K = \ln u/((3-D)s)$ with u harmonic. Take two point sources, $u_k = 1 + c_k|x - x_k|^{2-D}$ (for $D = 2$, $u_k = 1 + c_k \ln|x - x_k|$). By (A4), $K_1 + K_2$ must be a vacuum field, i.e. u_1u_2 must be harmonic. But

$$\nabla^2(u_1u_2) = 2\,\nabla u_1 \cdot \nabla u_2, \tag{11}$$

which is not identically zero for $x_1 \neq x_2$. Contradiction. □

Remark 1 (What the theorem does and does not show). The content of the theorem is that, with the spatial metric fixed by coincidence, the covariant static equation for the news count is linear in K only for $D = 3$. This is closely tied to the known dimension dependence of static reductions of general relativity (Sec. 6.1). Three

[1] The name was chosen by the author.

further points limit its weight. (a) Assumption (A4), exact superposition of counts, is strong: general relativity itself does not satisfy it, since its static two-body fields do not superpose. (b) Assumption (A3) was chosen partly because the alternative—news propagating with coincidence-weighted, non-covariant flux—gives $\beta = 0$ and a black-hole shadow of $8M$, both excluded by observations made in $D = 3$. The route is therefore a consistency condition on rules already adapted to three-dimensional data, not an independent derivation of $D = 3$. (c) The theorem is static; it does not establish that a dynamical theory with these properties exists (Sec. 7).

6.1 Precedent: the dimension dependence is familiar

The factor λ^{3-D} mirrors a known feature of static dimensional reduction in general relativity. A static metric in $D + 1$ dimensions written as

$$ds^2 = -e^{2U}\mathrm{d}t^2 + e^{-2U/(D-2)}\,\gamma_{ij}\,\mathrm{d}x^i\mathrm{d}x^j \tag{12}$$

decouples the lapse potential U from the spatial geometry γ_{ij}; in terms of the lapse $\lambda = e^U$ the spatial factor is $\lambda^{-2/(D-2)}$, which equals the inverse square of the lapse only when $D = 3$. The same structure underlies the Majumdar–Papapetrou solutions, whose higher-dimensional generalisations carry the exponent $1/(D - 2)$ in the spatial part [13]. The dimension dependence itself is therefore not new. What is new here, to our knowledge, is the reading of it as a condition for additivity of counts, given a spatial metric fixed by coincidence.

6.2 Consequence: the exponential metric

The metric (9) is not new: it was proposed by Papapetrou [14] and by Yilmaz [15], and Boonserm, Ngampitipan, Simpson and Visser showed that it describes a traversable wormhole with its throat at $r = M$ [16]. In the weak field it has $\beta = \gamma = 1$. It first differs from Schwarzschild at second post-Newtonian order: in isotropic coordinates general relativity has $g_{tt}g_{rr} = -(1 - M^2/4r^2)^2$ rather than -1. Its Ricci tensor is $R_{\mu\nu} = -2\,\partial_\mu U\partial_\nu U$ (with $U = M/r$), so in the language of general relativity it is sourced by a massless scalar with wrong-sign (phantom) energy.

For a non-rotating object the critical impact parameter of this metric (the shadow radius) is $b_c = 2e\,M \approx 5.4366\,M$, against $3\sqrt{3}\,M \approx 5.1962\,M$ for Schwarzschild, an increase of 4.63%. That difference is a property of the Papapetrou–Yilmaz geometry, shared with other exponential-metric proposals. It is not a prediction of the present framework: Sec. 7 shows that a propagating phantom with this normalisation is excluded by binary-pulsar timing, and no other completion that keeps the exponential exterior is in hand.

7 Discussion

7.1 What is established

- Route A is classical [1]. The substrate adds that the r^{2-D} law it relies on follows from the lineage mechanism, exactly at $\epsilon = 0$.
- Route B is a direct consequence of Pólya's theorem once the premise is granted that the gravitational potential is the discounted Green's function of random-walk lineages and that gravity is long-ranged. The forward process agrees with that Green's function. At the original run length the shells at $r \geq 6$ are noisy, with standard errors of order 10%; a run eight times longer is consistent with ratio 1 from $r = 1$ to $r = 16$. The screening length is $\xi = \sigma/\sqrt{2\epsilon}$. The exact lattice solution follows that Yukawa shape, to a few percent, from $r = 4$ to $r = 13$.
- Route C rests on the Mermin–Wagner theorem. Our simulations show that coincidence coupling realises it at noise ratio of order one: order is size-independent in $D = 3$ and decays in $D = 1, 2$, on lattices and on random graphs of matched degree. Keeping order in the presence of gravitational or random rate differences requires the covariant coupling of Eq. (8).
- Route D is a theorem under assumptions (A1)–(A4), for static fields and non-trivial gravity.

Routes B and C bound the dimension from below ($D \geq 3$), Route A from above ($D < 4$), and Route D gives $D = 3$ as a consistency condition. The criteria are not independent derivations: Routes A–C are known theorems applied to substrate mechanisms, and Route D rests on rules partly selected by three-dimensional observations. What the paper shows is that, given the stated premises, one substrate supports localised gravity, coherent quantum phase, bound structures and a consistent static counting closure only in three dimensions. None of this shows that $D = 3$ is dynamically inevitable.

7.2 What is not established

Premises and limitations of Routes B and C. Route B assumes that gravity is carried by random inheritance along lineages and is long-ranged; its $D = 2$ exclusion is only logarithmic, it does not cover a uniform matter density, and its transience argument is proved for lattices only. Route C assumes that quantum phase is carried by synchronised clocks with local coupling; a finite density of non-local coupling links restores order in $D = 2$; the $D = 2$ exclusion is quantitative only at noise ratio of order one; and macroscopic coherence cannot form by local synchronisation within the age of the universe, so it must be inherited.

The gravity sector. The simplest dynamical completion—a scalar news field—is excluded by observation in three independent ways. (a) It predicts no frame dragging, whereas Gravity Probe B measured a Lense–Thirring precession of $-37.2 \pm 7.2\,\mathrm{mas/yr}$ [18], excluding zero at 5.2σ. (b) Its preferred-frame parameters are $\alpha_1 = -8$, $\alpha_2 = -1$, against the limits $|\alpha_1| < 7 \times 10^{-5}$ and $|\alpha_2| < 2 \times 10^{-9}$ in Table 4 of Ref. [19]. (c) It radiates only a breathing mode, whereas LIGO/Virgo polarisation tests favour pure tensor over pure scalar polarisation [20]. The news must therefore carry the momentum of its source, i.e. gravity must be a tensor field.

A tensor news field composed multiplicatively reproduces the exponential metric statically and general relativity's first post-Newtonian physics, including frame dragging and transverse-traceless waves at leading order. It then faces Deser's self-consistency argument [21], whose unique completion is general relativity with a Schwarzschild exterior. A Lorentz-violating realisation in which the substrate frame acts as an aether requires a parameter value excluded by preferred-frame bounds [19].

The Lorentz-invariant reading that keeps the exponential metric is general relativity plus a phantom scalar of charge equal to the mass. The Einstein equation fixes the normalisation. With

$$T_{\mu\nu} = \varepsilon\Big[\partial_\mu\phi\,\partial_\nu\phi - \tfrac{1}{2}g_{\mu\nu}(\partial\phi)^2\Big], \tag{13}$$

$\phi = U = M/r$ and $\varepsilon = -1/(4\pi)$, one has $G_{\mu\nu} = 8\pi T_{\mu\nu}$ for every component outside the source. The static solution $\phi = M/r$ is the retarded solution of $\Box\phi = -4\pi\rho$. On a binary the scalar monopole is constant and the mass dipole vanishes in the centre-of-mass frame, at every mass ratio, so the leading radiation is quadrupole. The power leaving the source on a circular orbit is

$$P_{\mathrm{scalar}} = -\frac{16}{15}\frac{\mu^2 M^3}{a^5} = -\frac{1}{6}P_{\mathrm{GR}}, \tag{14}$$

where $P_{\mathrm{GR}} = (32/5)\mu^2 M^3/a^5$ is the Peters quadrupole. The sign is the phantom's: the scalar flux carries negative energy. The tensor quadrupole is unchanged at this order, so the orbital energy changes at $-(5/6)P_{\mathrm{GR}}$. The inspiral is slower than in general relativity by one sixth; it does not reverse. A spectral average on one Keplerian period reproduces Eq. (14) to a part in 10^6 and moves the ratio only to -0.168 at eccentricity 0.8. The trace, or breathing, share of that scalar power is below one percent up to that eccentricity.

Binary pulsars exclude the result. The double pulsar validates general relativity's quadrupolar decay at 1.3×10^{-4} (95% confidence) [22]; Eq. (14) predicts 0.833 times that decay at $e = 0.088$. For PSR B1913+16 the ratio of the observed orbital decay to the general-relativistic prediction is 0.9983 ± 0.0016 [23], against 0.832 at $e = 0.617$. A scalar mass large enough that the double-pulsar frequency does not propagate has Yukawa range $c/\Omega \simeq 1.4\,\mathrm{AU}$ and would screen the Newtonian potential inside the solar system. No massive or non-radiating completion that preserves assumptions (A1)–(A4) is constructed here.

The strong-field reading of the theorem does not survive. The shadow excess and the absence of a horizon remain properties of the Papapetrou–Yilmaz metric. What remains is the static counting statement: under (A1)–(A4) a non-trivial vacuum is consistent only for $D = 3$, and the exterior agrees with general relativity through first post-Newtonian order. A non-radiating lapse sector would have to abandon (A3), and with it the proof as stated.

The same composition law, kept to first order in the spin, multiplies the harmonic-gauge gravitomagnetic shift by $\sinh(2M/r)/(2M/r) = 1 + \frac{2}{3}(M/r)^2 + \cdots$. The first correction is of order seven when the Newtonian potential is counted as order two and the shift itself as order three. Gravity Probe B measures the linear term, which this law shares with general relativity. At the surface of the Earth the fractional correction is of order 10^{-19}. At $r = M$ the factor is $\sinh 2/2 \approx 1.81$. That factor is a property of the composition law excluded by Eq. (14). The stationary vacuum exterior consistent with the pulsar test is the Kerr metric.

Other open questions. Matter is modelled only through test particles and simple composites; the parameters ϵ, δ, α (hence Newton's constant and the screening length) are not derived; and independent review of the code and of the novelty claims is still needed.

7.3 Summary

In a substrate built only from clocks, ticks and coincidences, several known dimension criteria—stable bound orbits, localised gravity, coherent quantum phase—arise together, and the static counting rules close consistently only in three spatial dimensions. Given the stated premises, these are jointly satisfied only for $D = 3$. Each route has limitations. The calculation above excludes a propagating phantom with charge equal to mass, so Route D does not supply a strong-field alternative to the Schwarzschild exterior. The linear-spin factor of the same composition law is a property of that excluded law.

Acknowledgements

The computations, literature checks and drafting of this manuscript were assisted by an AI system (Claude, Anthropic). The author is responsible for the content and for any errors.

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